Effect of flow on the quasiparticle damping rate in hot QCD plasma

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Abstract

We derive an expression for the quark damping rate in hot quark gluon plasma in presence of flow. Unlike earlier treatments here all the bath particles are out of equilibrium due to the existence of non-zero velocity gradient. The quasi particle damping rate is found to be finite once the hard thermal loop (HTL) corrected gluon propagator is used.

In recent years several authors have calculated the quasiparticle damping rate (Γ) both in quantum chromodyamic (QCD) and quantum electrodynamic (QED) plasmas [1, 2, 3, 4, 5, 6, 7]. The calculations are important to understand various dynamical properties of such plasma associated with the quasiparticle excitations. Such investigations are important as the inverse of the damping rate gives the relaxation time used in the collision term of the Boltzmann equation. Therefore, once known, it can be used to estimate the thermalization time of a preequilibrium parton gas in relativistic heavy ion collision [8]. The expression of damping rate can also be used to derive the drag and the diffusion coefficients of the quasiparticle by folding the expression with relevant energy or momentum transfer [9, 10, 11, 12, 13, 14, 15].

For the relativistic hot plasma, it has been observed that quasi-particle life time or the damping rate calculations are plagued with infrared divergences. In non-relativistic plasma, where one considers only the coulomb or electric interaction such divergences are removed by the Debye screening effects. The problem becomes non-trivial in dealing with relativistic plasma where one has to worry about the electric and transverse interactions both. This additional complication actually arises due to the absence of static magnetic screening. In case of QED, in [6, 7] the authors have shown that the electric contribution to the quasi-particle damping rate with plasma screening effects Γ is finite and is of the order $\Gamma_{long} \sim g^2 T$ whereas the transverse part remains divergent. This is because the later is only dynamically screened. To obtain a finite result authors had to develop another resummation scheme to obtain finite result [6, 7]. In case of QCD one also encounters similar problem in hot plasma which however can be removed by assuming a 'magnetic mass' for the gluons. It would be worthwhile to note that in degenerate plasma one obtains finite results with magnetic interactions without further resummation unlike its high temperature counterpart. For detailed discussions about these issues we refer the readers to [6, 7, 16, 17].

The purpose of the present work is to calculate the quark damping rate in QCD plasma with flow. Naturally this is a major departure from the above cited calculations where always the bath particles are assumed to be in equilibrium. This is true only when there exists no velocity or temperature gradient in the plasma and there is no external force. The scenario that we currently consider is different. Here, we assume that there exists a non-zero velocity gradient *i.e* flow in the plasma. Under such condition, one can no longer assume that the bath particles follow the ordinary equilibrium Bose-Einstein or Fermi-Dirac distributions for the interacting bosons or fermions respectively. The situation is very similar to what one encounters for the calculation of transport quantities like coefficient of viscosity or conductivity etc. We shall see in this letter that the transferred momentum dependence of the damping rate in presence of flow is very different leading to results which are finite once the plasma effects are included via the dressed propagator.

To expose this further, we first recall that, the calculation of decay width in plasma is directly related to the collision integral of the Boltzmann equation given by the following equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}.\nabla_{\mathbf{r}} + \mathbf{F}.\nabla_{\mathbf{p}}\right) n_p = -C[n_p], \tag{1}$$

here, $\mathbf{p}$ is the momentum of the quasiparticle, $\mathbf{F}$ is the external force. The right hand side of the equation is the collision integral. For the relativistic plasma, one assumes, that all the particles are moving at the speed of light, which means

that the velocity is a unit vector, $\mathbf{v_p} = \hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. For two body interaction, which we assume here, the collision integral can be written as,

$$C[n_p] = \int \frac{d^3k}{(2\pi)^3}\frac{d^3p^{'}}{(2\pi)^3}\frac{d^3k^{'}}{(2\pi)^3} \times n_p n_k(1\pm n_p^{'})(1\pm n_k^{'}) - n_p^{'} n_k^{'}(1\pm n_p)(1\pm n_k)(2\pi)^4\delta^4(p+k-p^{'}-k^{'})|M|^2, \tag{2}$$

n_i are the Fermi and Bose quasiparticle distribution functions for quarks and gluons and the $\pm$ includes stimulated emission or Pauli blocking. $|M|^2$ is the relevant squared matrix element for the $2 \to 2$ processes summed over final states and averaged over initial states.

In absence of the external force and gradients of temperature, velocity or density Eq.(1) takes very simple form. If n_p is considered to be slightly out of equilibrium *i.e* $n_p = n_p^{eq} + \delta n_p$, in relaxation time approximation the solution of the Eq.(1) can be written as $n_p(t) = n_p(0)\mathrm{e}^{-\Gamma_p t}$, where Γ_p is the damping rate of the quark moving with the momentum $\mathbf{p}$ related to the inverse life time τ_p^{-1}. Mathematically,

$$C[n_p] = \delta n_p/\tau_p, \tag{3}$$

where, τ_p is the relaxation time of the collision in this case and $1/\tau_p = \Gamma_p$. The collision term $C[n_p]$ consists of two parts. One involving equilibrium distribution function and the other non-requilibrium δn_p's,

$$C[n_p] = C^{eq}[n_p] + C^{non-eq}[n_p]. \tag{4}$$

The $C^{eq}[n_p]$ vanishes due to the energy conserving delta function *i.e*,

$$[n_p n_k(1-n_{p'})(1-n_{k'}) - n_{p'}n_{k'}(1-n_p)(1-n_k)]\delta(p+k-p^{'}-k^{'}). \tag{5}$$

From the $C^{non-eq}[n_p]$ part we get the following expression for the damping-rate when only one particle is slightly away of equilibrium [5, 6, 7],

$$\Gamma(p) = \int_{k,p',k'} \left[n_k(1-n_{k'})(1-n_{p'}) - n_{p'}n_{k'}(1-n_k)\right](2\pi)^4\delta^4(p+k-p^{'}-k^{'})|M|^2. \tag{6}$$

We use $\int_{k,p,p'}$ instead of $d^3kd^3p^{'}d^3k^{'}/(2\pi)^9$ throughout the letter. In the calculation of the above damping rate only the test particle is assumed to be slightly away from equilibrium while the bath particles are not. In this case we do not need to know about the explicit form of the non-equilibrium distribution function as it gets canceled from both sides in the Eq.(3). On the other hand the general expression for the damping rate when four particles are out of equilibrium is,

$$\begin{aligned}\Gamma(p) = &\Bigg(\int_{k,p,p'} \Big[\delta n_p\left(n_k(1\pm n_{p'})(1\pm n_{k'}) \mp n_{p'}n_{k'}(1\pm n_k)\right) \\ &+\delta n_k\left(n_p(1\pm n_{p'})(1\pm n_{k'}) \mp n_{p'}n_{k'}(1\pm n_p)\right) - \delta n_{p'}\left(n_{k'}(1\pm n_p)(1\pm n_k) \mp n_p n_k(1\pm n_{k'})\right) \\ &-\delta n_{k'}\left(n_{p'}(1\pm n_p)(1\pm n_k) \mp n_p n_k(1\pm n_{p'})\right)\Big](2\pi)^4\delta^4(p+k-p^{'}-k^{'})|M|^2\Bigg)/\delta n_p.\end{aligned} \tag{7}$$

In the above equation it is evident that if we put $\delta n_k = \delta n_{p'} = \delta n_{k'} = 0$, then we get back the Eq.(6). To obtain the expression of the Eq.(7) we have to know the explicit form of δn_i $(i = p, k, p^{'}, k^{'})$ unlike the above situation. The non-equilibrium distribution function takes different form depending upon the problem considered. For our purpose we take the distribution function as [18, 19, 20, 21],

$$\delta n_i = C\frac{\eta}{s}\frac{\partial n_i}{\partial p_i}\Phi_{i,xy}\mathcal{X}_{xy} \tag{8}$$

where, $\Phi_{i,xy} = \hat{p}_{i,x}\hat{p}_{i,y} f(p/T)$ and C is some constant which has been estimated in [21]. In the presence of a small shear flow $u(y)$ in the x direction $\mathcal{X}_{xy} = \partial u_x/\partial y$, $f(p/T)$ is some rotationally invariant function depending only on the energy of the excitation which has to be determined from the variational calculation of the Boltzmann equation [18, 19, 20].

But a much simpler and standard way is to take the trial function in the viscous process as, $f(p/T) = (p/T)^2$ [19]. Substituting the non-equilibrium distribution function Eq.(8) in the collision integral we obtain,

$$\begin{aligned} C_{\Phi_{xy}}(p) &= (-)\frac{C\eta}{sT}\int_{k,p,p'} n_p n_k (1 \pm n_p^{'})(1 \pm n_k^{'})|M|^2 (2\pi)^4 \delta^4(p + k - p^{'} - k^{'}) \\ &[\Phi_{xy}(\mathbf{p}) + \Phi_{xy}(\mathbf{k}) - \Phi_{xy}(\mathbf{p}^{'}) - \Phi_{xy}(\mathbf{k}^{'})]. \end{aligned} \tag{9}$$

To proceed further we introduce the following expression here,

$$\begin{aligned} [\Phi_{xy}, C_{\Phi_{xy}}] &= (-)\frac{C\eta}{sT}\int_{k,p,p'} n_p n_k (1 \pm n_p^{'})(1 \pm n_k^{'})|M|^2 (2\pi)^4 \delta^4(p + k - p^{'} - k^{'}) \\ &[\Phi_{xy}(\mathbf{p}) + \Phi_{xy}(\mathbf{k}) - \Phi_{xy}(\mathbf{p}^{'}) - \Phi_{xy}(\mathbf{k}^{'})]^2. \end{aligned} \tag{10}$$

We also define,

$$[\Phi_{xy}, S_{xy}] = (-)\frac{C\eta}{sT} n_p (1 \pm n_p) f^2(p/T), \tag{11}$$

where, $S_{xy} = \delta n_p$. With the help of above two equations one can easily check that expression for the damping rate takes the following form,

$$\Gamma(p) = \frac{[\Phi_{xy}, C_{\Phi_{xy}}]}{[\Phi_{xy}, S_{xy}]}. \tag{12}$$

In the collision integral we use the spatial delta function to perform the $\mathbf{k}^{'}$ integration and to shift the $\mathbf{p}^{'}$ integration into an integration over $\mathbf{q}$, where, $\mathbf{q} = \mathbf{p}^{'} - \mathbf{p}$. Hence,

$$\begin{aligned} \Gamma(p) &= \frac{T^4}{(2\pi)^4 p^4}\int_0^\infty q^2 dq k^2 dk \int_{-1}^{1} d\cos\theta d\cos\theta_{kq} \int_0^{2\pi} d\phi n_k (1 \pm n_k^{'})|M|^2 \delta(p + k - p^{'} - k^{'}) \\ &[\Phi_{xy}(\mathbf{p}) + \Phi_{xy}(\mathbf{k}) - \Phi_{xy}(\mathbf{p}^{'}) - \Phi_{xy}(\mathbf{k}^{'})]^2. \end{aligned} \tag{13}$$

It is convenient to introduce a dummy integration variable ω,

$$\delta(p + k - p^{'} - k^{'}) = \int_{-\infty}^{\infty} d\omega \delta(\omega + p - p^{'})\delta(\omega + k - k^{'}). \tag{14}$$

Evaluating $\mathbf{q} = \mathbf{p}^{'} - \mathbf{p}$ in terms of p, q and $\cos\theta_{pq}$ and defining $t = \omega^2 - q^2$ we find,

$$\begin{aligned} \delta(\omega + p - p^{'}) &= \frac{p^{'}}{pq}\delta\left(\cos\theta_{pq} - \frac{\omega}{q} - \frac{t}{2pq}\right) \\ \delta(\omega + k - k^{'}) &= \frac{k^{'}}{kq}\delta\left(\cos\theta_{kq} - \frac{\omega}{q} + \frac{t}{2kq}\right). \end{aligned} \tag{15}$$

Now, to proceed further square bracketed term containing Φ_{xy} in the Eq.(13) has to be evaluated. Considering the small angle scattering limit which is a very reasonable assumption in the present context [20], one can easily obtain the equation mentioned below. We need not worry about the cross terms since they do not exist after carrying out the integrations over $d\omega$ and $d\phi$. So, the terms which actually contribute to the damping rate are the following,

$$\begin{aligned} [\Phi_{xy}(\mathbf{p}) - \Phi_{xy}(\mathbf{p}^{'})]^2 &= \omega^2 [f(p)']^2 + 3\frac{q^2 - \omega^2}{p^2}[f(p)]^2, \\ [\Phi_{xy}(\mathbf{k}) - \Phi_{xy}(\mathbf{k}^{'})]^2 &= \omega^2 [f(k)']^2 + 3\frac{q^2 - \omega^2}{p^2}[f(k)]^2. \end{aligned} \tag{16}$$

Upto now, we have not made any reference to the interaction which is contained in the matrix amplitude. For QCD plasma, to calculate quark damping rate we need to consider $qq \to qq$ as well as $qg \to qg$ processes. For the former case the squared matrix element is given by,

$$|M_{qq}|^2 = \frac{4}{9} g^4 \frac{u^2 + s^2}{t^2}. \tag{17}$$

The above matrix amplitude squared gives divergent contribution to the damping rate due to the presence of singularity in t^{-2}. Hence, one needs to incorporate the screening effect. In case of small momentum transfer, $q << p$, $k \sim T$, from the energy conservation delta function we obtain $\omega = p - p^{'} \simeq \mathbf{v_p}.\mathbf{q} = -\mathbf{v_k}.\mathbf{q}$. The velocity projections transverse to q have lengths $|\mathbf{v_{p.T}}| = |\mathbf{v_{k,T}}| = \sqrt{(1-x^2)}$, where, $x = \omega/q$. Consequently, $|\mathbf{v_{p.T}}|.|\mathbf{v_{k.T}}| = (1-x^2)\cos\phi$, where, ϕ is the angle between the two velocity projections on q. For $q << T$ we have,

$$\begin{aligned} s \simeq -u \quad & \simeq 2pk(1-\cos\theta_{pk}) \\ & \simeq 2pk(1-x)(1-\cos\phi) \end{aligned} \tag{18}$$

Hence, the expression in Eq.(17) takes the following form,

$$|M_{qq}|^2 = g^4\frac{4}{9}\left[\frac{1}{q^4} - \frac{(1-x^2)^2\cos^2\phi}{(q^2-\omega^2)^2}\right]. \tag{19}$$

From the above expression it is evident that the matrix amplitude squared is infrared divergent. To circumvent this problem one adopts Braaten and Pisarski's method [22]. In this formalism, the interaction is split into hard and soft momentum transfer domain by introducing an intermediate momentum scale q^* [23]. For the hard sector one then uses the bare propagator, for the soft sector, on the other hand, one loop corrected gluon propagator is used. To get the final result both contributions have to be added up and on addition the arbitrary momentum scale q^* gets canceled.

The dressed gluon propagator involves the longitudinal and transverse polarization functions which are given by:

$$\begin{aligned} \Pi_L(q,\omega) \quad & = m_D^2\left[1 - \frac{x}{2}\ln\left(\frac{x+1}{x-1}\right)\right], \\ \Pi_T(q,\omega) \quad & = m_D^2\left[\frac{x^2}{2} + \frac{x(1-x^2)}{4}\ln\left(\frac{x+1}{x-1}\right)\right], \end{aligned} \tag{20}$$

where, m_D is the Debye mass. Further simplifications can be made if small angle approximation is made which gives $\Pi_L \simeq m_D^2$ and $\Pi_T \simeq i(\pi/4)xm_D^2$.

With the dressed propagator for the soft sector the matrix amplitude squared takes the following form,

$$|M_{qq}|^2 = g^4\frac{4}{9}\left[\frac{1}{(q^2+\Pi_L)^2} + \frac{(1-x^2)^2\cos^2\phi}{(q^2-\omega^2+\Pi_T)^2}\right]. \tag{21}$$

It is to be noted that the cross term of the longitudinal and the transverse interaction vanishes after the $d\phi$ integration, hence, we omit the term here.

Using the expressions of the Eq.(16) and the quark-quark scattering matrix amplitude square in the Eq.(13) for the electric interaction in the soft region we have,

$$\begin{aligned} \Gamma_{long}^{soft}(p) \quad & = g^4\left(\frac{5T^3}{81\pi p^2} + \frac{7\pi T^5}{81p^4}\right)\int_0^{q^*}\frac{q^3dq}{(q^2+m_D^2)^2} \\ & = g^4\left(\frac{5T^3}{81\pi p^2} + \frac{7\pi T^5}{81p^4}\right)\left[\log\left|\frac{q^*}{m_D}\right| - \frac{1}{2}\right]. \end{aligned} \tag{22}$$

In case of the momentum region where $q > q^*$ we can ignore the term containing Debye mass in the denominator, hence, one finds for the damping rate,

$$\Gamma_{long}^{hard}(p) = g^4\left(\frac{5T^3}{81\pi p^2} + \frac{7\pi T^5}{81p^4}\right)\left[\log\left|\frac{q_{max}}{q^*}\right|\right]. \tag{23}$$

On addition of the Eqs.(22) and (23) the intermediate momentum scale q^* gets canceled. Hence, we obtain a finite result in the longitudinal sector independent of the arbitrary momentum scale,

$$\Gamma_{long}(p) = g^4\left(\frac{5T^3}{81\pi p^2} + \frac{7\pi T^5}{81p^4}\right)\left[\log\left|\frac{q_{max}}{m_D}\right| - \frac{1}{2}\right]. \tag{24}$$

In case of the transverse interaction one can proceed in the same way like the longitudinal one and obtain the final result,

$$\Gamma_{trans}(p) \qquad = g^4\left(\frac{5T^3}{162\pi p^2}+\frac{7\pi T^5}{162p^4}\right)\left[\log\left|\frac{2q_{max}}{\sqrt{\pi}m_D}\right|+\frac{7}{15}\right]. \tag{25}$$

Hence, the final expression for the damping rate in presence of flow in the medium has the following form,

$$\Gamma(p) = g^4\left(\frac{5T^3}{54\pi p^2}+\frac{7\pi T^5}{54p^4}\right)\left[\log\left|\frac{2}{\sqrt{\pi}}\right|+2\log\left|\frac{q_{max}}{m_D}\right|-\frac{1}{30}\right]. \tag{26}$$

From the kinematics the q_{max} here can be chosen as T [19]. From the final expression it is evident that at high temperature in presence of flow in the medium quasiparticle damping rate turns out to be finite and logarithmic in nature. Interesting to note here that the viscous relaxation time also exhibits similar logarithmic behaviour [5, 18, 19]. This is not surprising as these quantities are related to each other upto an integral [24].

To summarize in the present work we have derived an expression for the quark damping rate in QCD plasma in presence of flow. It has been shown that the quark damping rate in presence of a velocity gradient differs from the usual damping rate because of the presence of $\Phi_{xy}(p)$ terms. The latter, brings an extra ω^2 term in the numerator which changes the infrared behaviour of Γ completely. Interestingly, here we find that the quasi-particle damping rate turns out to be finite without further resummation as mentioned in the introduction. It is important to note also that Γ in the present treatment is independent of the coefficient of viscosity η and the magnitude of the flow gradient, although the distribution functions contain both the terms.

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